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Low Rank Approximation For Entangled Quantum Systems

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Introduction to Quantum Entanglement and Mathematical Models

Quantum entanglement is a marvelous phenomenon that plays an essential task in quantum informatics [1, 2, 3, 4, 5, 6]. The miracle demonstrates how pairs or groups of particles generate, interact, or share properties so that the variation of one particle will instantly transform the characteristics of another particle. In other words, we could imagine that there is a mysterious communication channel which forwards information even faster than the speed of light. To the study of entanglement, we suitably represent quantum states in terms of some properly selected basis over the complex field. Then, we express the rule of entanglement in quantum mechanics systems as Kronecker products between states of density matrices of the subsystems. Subsequently, we can investigate the property of entanglement as the following low rank approximation problem:

$$\min_{\substack{\theta_r \in R_+, x_r \in C^m, \; y_r \in C^n \\ \|x_r\|=1, \|y_r\|=1}} \left\| \rho - \sum_{r=1}^{k} \theta_r (x_r x_r^*) \otimes (y_r y_r^*) \right\|_F^2$$

①

where * denotes the conjugate transpose, stands for the Kronecker product, and ρ $C^{mn\times mn}$ is a positive definite matrix.

Difficulties and Breakthrough

A low rank approximation to an entangled bipartite system represented in ① is fundamentally different from a conventional tensor approximation with several new challenges:

The twist caused by the Kronecker product destroys the multi-linearity. The famous alternating least squares techniques can hardly be applied.

To correctly characterize the quantum properties, it is necessary to involve complex variables. The approximation amounts to the optimization of real-valued functions over the complex spaces.

The approximation needs to deal with a proper low rank, which is not known a priori, and to maintain the probability distribution among the states.

Thus, we propose a dynamical system approach to tackle the problem ① with complex variables directly. This method, utilizing the projected gradient flow and the notion of Wirtinger calculus, is concise and can achieve convergence from any starting point. We ensure for no difficulty that the requirement of the combination coefficients θ_r's must be a probability distribution. When needed, we can even obtain the desired low-rank approximation by dynamically adjusting the predicted rank k.

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