

A Review of Semi-Analytical Numerical Methods for Laminated Composite and Multilayered Functionally Graded Elastic/Piezoelectric Plates and Shells

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This article was intended to present a comprehensive survey regarding the papers examining various three-dimensional (3D) structural behavior of laminated composite (LC) plates/shells and functionally graded (FG) plates/shells using a semi-analytical numerical method. This review article contains 198 references, of which 50 papers are contributed from the research group of the first author, Professor Chih-Ping Wu.

Due to the fact that the pure 3D analytical approaches, including the state space, the series expansion, the modified Pagano, and the perturbation methods, are mathematically complicated and that the solution processes of the pure 3D numerical methods, including the 3D differential quadrature (DQ), the 3D finite element (FE), and the 3D differential reproducing kernel (DRK) meshless methods, are very time consuming, a class of compromise approaches, so-called 3D semi-analytical numerical method, has thus been developed for the quasi-3D structural behavior analysis of these LC and FG plates/shells.



The first author's research group incorporated the state space and the perturbation methods with the DQ, the FE, and the DRK methods to develop some innovative computational methods for the above-mentioned structures, such as the state space DQ, the state space DRK, the asymptotic DQ, the asymptotic DRK, and the asymptotic FE methods, most of which were proposed for the first time in the literature.

The accuracy and the convergence rate of Wu's semi-analytical DQ, DRK, and FE methods were validated by comparing their solutions with the exact 3D solutions available in the literature. The results showed the solutions of these semi-analytical numerical methods are in excellent agreement with the exact 3D solutions and the convergence rate of these semi-analytical numerical methods is rapid. The detailed comparative study of the results obtained using different semi-analytical numerical methods can be found in the article ([Composite Structures, vol. 147, pp. 1-15](#)).

Table 1. Comparisons with regard to the meshes and configuration between the current finite cylindrical prism and traditional layerwise shell elements.

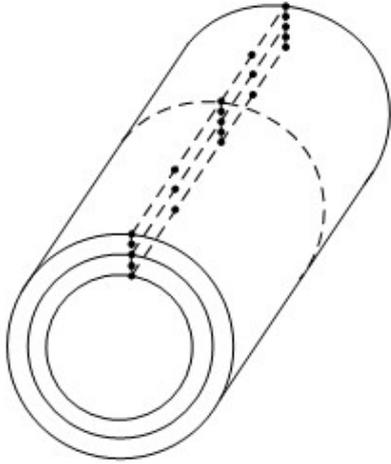
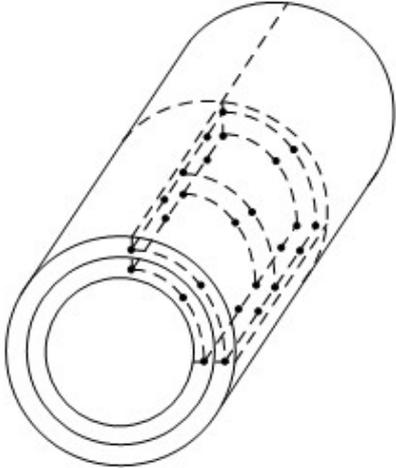
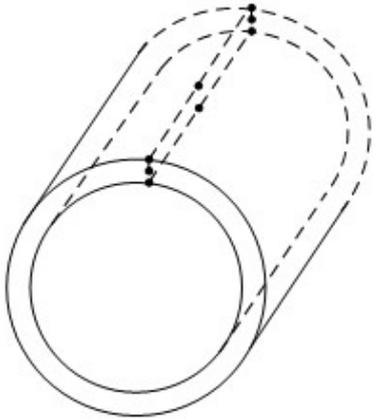
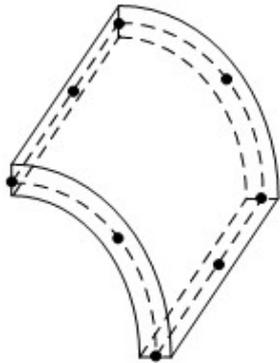
Elements	Finite cylindrical prism elements	Layer-wise shell elements
Meshes	 <p>(the 2x1x2 mesh)</p>	 <p>(1/4 cylindrical model with a 2x1x2 mesh)</p>
Configuration	 <p>(a Q8 prism element)</p>	 <p>(a Q8 shell element)</p>

Table 2. Comparative studies of the results obtained by using the current finite prism method and the exact 3D solutions for the stress and deformation analyses of a simply-supported, [90°/0°/90°] laminated composite cylinder under a sinusoidally distributed load ($L/R=4$ and $R/h=4$).

Theory	Related parameters	$\bar{\sigma}_x\left(\frac{L}{2}, 0, \frac{h}{2}\right)$	$\bar{\sigma}_\theta\left(\frac{L}{2}, 0, \frac{h}{2}\right)$	$\bar{\tau}_{x\theta}\left(0, \frac{\pi}{2}, \frac{h}{2}\right)$	$\bar{u}_r\left(\frac{L}{2}, 0, 0\right)$
RMVT-based Q8 FCPM	Mesh (36x6)	0.1238	6.5335	0.1080	4.0056
	Mesh (48x9)	0.1260	6.5415	0.1081	4.0088
	Mesh (48x12)	0.1262	6.5433	0.1081	4.0086
RMVT-based Q9 FCPM	Mesh (36x6)	0.1238	6.5333	0.1080	4.0056
	Mesh (48x9)	0.1256	6.5414	0.1081	4.0088
	Mesh (48x12)	0.1262	6.5432	0.1081	4.0086
Modified Pagano method		0.1269	6.5462	0.1082	4.0097
3D Elasticity Solutions		0.1270	6.5450	0.1081	4.0090