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H^∞ Robust Interval Fuzzy Control of a Class of Uncertain Chaotic System

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Over the past two decades, chaotic phenomena have been discussed extensively in various fields of science such as in circuit systems, quantum systems, chemical systems, and etc. To obtain a precise mathematical model for an uncertain chaotic system is very difficult because environmental factors act upon system parameters. However, the uncertainty and disturbances to the parameters can be represented by interval numbers. The T-S fuzzy modeling method is to offer an alternative approach to describing an uncertain chaotic system. For the interval fuzzy system, each local model is described by the linear interval model. Thus, we want to point out results that show where Theorem 1 is different from that of the ordinary T-S fuzzy control system.

We propose the interval fuzzy model where the local dynamics in different state space regions are represented by corresponding linear interval models. The overall model of the fuzzy system is achieved by fuzzy blending of these local linear interval models. We design the linear feedback controller for each local linear interval model. The resulting overall fuzzy controller, which is nonlinear in general, is again a fuzzy blending of each individual linear controller. As for the stability, H^∞ performance, and design issues of the interval control system, we exploit LMI methodologies. The LMI based design plays a very important role in analyzing the stability and in synthesizing the nonlinear controller for the interval fuzzy model.

Consider an uncertain chaotic system as

$$\dot{x}(t) = f(\mathrm{a}(t);x) + g(\mathrm{b}(t);x)\cdot u + w(t) \tag{1}$$

where $x(t)\in\Re^n$ denotes the state vector assumed to be accessible, $u(t)\in\Re^m$ is the control input, $w(t)\in\Re^{n\times 1}$ denotes the unknown but bounded disturbance, $\mathrm{a}(t)\in\Re^{n_a}$ and $\mathrm{b}(t)\in\Re^{m_b}$. Suppose $a(t)$ and $b(t)$ are time-varying vectors with bounds a, a, b and b, respectively.

Now, the interval fuzzy system is introduced to model the uncertain chaotic system (1). Premise variables of the interval fuzzy system might be chosen by the nonlinear terms $f(a(t);x)$ and $g(b(t);x)$. We can set up the interval model as follows.

Plant Rule k:

IF $z_{10}(t)$ is M_k^1 and … and $z_{p0}(t)$ is M_k^p, THEN $\dot{x}(t) = A_k x(t) + B_k u(t) + w(t)$, $k=1,2,...,r$ (2)

where

$$A_k = \{[a_{kij}]\} \in \mathbf{A}_k = [\underline{A}_k, \bar{A}_k] = \{[\underline{a}_{kij}, \bar{a}_{kij}] : \underline{a}_{kij} \le a_{kij} \le \bar{a}_{kij}, 1 \le i, j \le n\} \tag{3a}$$

$$B_k = \{[b_{kil}]\} \in \boldsymbol{B}_k = [\underline{B}_k, \bar{B}_k] = \{[\underline{b}_{kil}, \bar{b}_{kil}] : \underline{b}_{kil} \le b_{kil} \le \bar{b}_{kil}, 1 \le i \le n, 1 \le l \le m\} \tag{3b}$$

are interval matrices, M_k^j is the fuzzy set, r is the number of IF-THEN rules, and $z_{10}(t) \ldots z_{p0}(t)$ are premise variables. The overall interval fuzzy model of (2) is inferred as follows.

$$\dot{x} = \sum_{k=1}^{r} \mu_k(z_0)(A_k x + B_k u) w(t) \tag{4}$$

where $\mu_k(z_0) = \omega_k(z_0) / \sum_{k=1}^{r} \omega_k(z_0)$, $z_0 = [z_{10} \cdots z_{p0}]$, $\sum_{k=1}^{r} \omega_k(z_0) > 0$ and $\omega_k(z_0) = \prod_{j=1}^{l} M_k^j(z_0) \ge 0$, $k=1,2,...,r$, for all t. $M_k^j(z_0)$ is the grade of membership of z_{j0} in M_k^j. From (4) we have $\sum_{k=1}^{r} \mu_k(z_0) = 1$ and $\mu_k(z_0) \ge 0$. Applying the expression to (4), one can obtain

$$A_k = \{[a_{kij}^0 + \Delta a_{kij}]\} = A_{k0} + \sum_{i,j=1}^{n} e_i (\delta_{kij} a_{kij}^0) e_j^T, \left|(\delta_{kij} a_{kij}^0)\right| = \left|\Delta a_{kij}\right| \le \Delta \bar{a}_{kij} \tag{5a}$$

$$B_k = \{[b_{kij}^0 + \Delta b_{kij}]\} = B_{k0} + \sum_{i=1}^{n} \sum_{j=1}^{m} h_i (\sigma_{kij} b_{kij}^0) h_j^T, \left|(\sigma_{kij} b_{kij}^0)\right| = \left|\Delta b_{kij}\right| \le \Delta \bar{b}_{kij} \tag{5b}$$

where $e_i \in \Re^n$ or $h_i \in \Re^m$ denotes the column vector with the i-th element to be 1 and the others to be 0.

The PDC concept is utilized to design the overall fuzzy controller which is achieved by integration of all local state feedback fuzzy controllers in each fuzzy rule. The rules of the fuzzy controller are described as follows.

Controller Rule k:

IF $z_{10}(t)$ is M_k^1 and … and $z_{p0}(t)$ is M_k^p,

THEN $u(t)=F_k x(t)$, $k=1,2,...,r$ (6)

where F_k presents the state feedback gain matrices. Then, the final composite state feedback fuzzy controller is obtained as

$$u(t) = \sum_{k=1}^{r} \mu_k F_k x(t) \tag{7}$$

Substituting (7) for (4) yields the closed-loop interval fuzzy control system

(8)

$$\dot{x}=\sum_{k=1}^{r}\sum_{s=1}^{r}\mu_k\mu_s\left\{\left(A_k+B_kF_s\right)x\right\}+w(t)$$

Since H^∞ control is a very important control design to efficiently eliminate the effect of $w(t)$ on an uncertain chaotic system, it will be employed to deal with the robust performance control in (4). To consider the following H^∞ control performance:

$$\int_0^{t_f}x^T(t)Qx(t)dt<x^T(0)Px(0)+\rho^2\int_0^{t_f}w^T(t)w(t)dt \tag{9}$$

where t_f is the terminal time of control, ρ is the prescribed attenuation level, the symmetric positive-definite weighting matrix Q is specified beforehand according to the design purpose, and P is some symmetric positive-definite weighting matrix.

We choose a Lyapunov function for the uncertain chaotic system (1) as

$$V(t)=x^T(t)Px(t) \tag{10}$$

where the weighting matrix $P=P^T>0$ and the time derivative of $V(t)$ is

$$\dot{V}(t)=\dot{x}^T(t)Px(t)+x^T(t)P\dot{x}(t) \tag{11}$$

Substituting equation (7) and (5)for (11) implies

$$\begin{aligned}\dot{V}=&\sum_{k=1}^{r}\sum_{s=1}^{r}\mu_k\mu_s x^T\times\left\{P\left[A_{k0}+B_{k0}F_s\right]+\left[A_{k0}+B_{k0}F_s\right]^T P\right\}x(t)\\&+\sum_{k=1}^{r}\sum_{s=1}^{r}\mu_k\mu_s x^T(t)\left\{\sum_{i,j=1}^{n}Pe_i\left(\delta_{kij}a_{kij}^0\right)e_j^T+\sum_{i,j=1}^{n}\left[e_i\left(\delta_{kij}a_{kij}^0\right)e_j^T\right]^T P\right\}x(t)\\&+\sum_{k=1}^{r}\sum_{s=1}^{r}\mu_k\mu_s x^T(t)\left\{\sum_{i=1}^{n}\sum_{l=1}^{m}Pe_i\left(\sigma_{kil}b_{kil}^0\right)h_l^T F_s+\sum_{i=1}^{n}\sum_{l=1}^{m}\left[e_i\left(\sigma_{kil}b_{kil}^0\right)h_l^T F_s\right]^T P\right\}x(t)\\&+w^T(t)Px(t)+x^T(t)Pw(t)\end{aligned} \tag{12}$$

Theorem 1. If there exists a symmetric positive-definite matrix $X=P^{-1}\in\Re^{n\times n}$, matrices $Z_k,\ Z_s\in\Re^{m\times n}$, and positive real scalars λ_{kij}, λ_{sij}, α_{kil} and α_{sil} ($i,j=1,2,...,n$, $l=1,2,...,m$, $k,s=1,2,...,r$) satisfy the following LMIs

$$\begin{bmatrix}G & U_1 & U_2 & X\\ U_1^T & -V_1 & 0 & 0\\ U_2^T & 0 & -V_2 & 0\\ X & 0 & 0 & -Q^{-1}\end{bmatrix}<0 \tag{13}$$

where

$$G_1=XA_{k0}^T+A_{k0}X+Z_k^T B_{k0}^T+B_{k0}Z_k+\sum_{i,j=1}^{n}\lambda_{kij}\Delta\bar{a}_{kij}^2 e_i e_i^T+\sum_{i=1}^{n}\sum_{l=1}^{m}\alpha_{kil}\Delta\bar{b}_{kil}^2 e_i e_i^T+\frac{1}{\rho^2},$$

$U_1 = \Big[\underbrace{X \quad \cdots \quad X}_{n}\Big] \in \Re^{n \times n^2}$, $U_2 = \Big[\underbrace{Z_s^T \quad \ldots \quad Z_s^T}_{n}\Big] \in \Re^{n \times nm}$, $V_1 = diag\{\lambda_{k11} \quad \cdots \quad \lambda_{knn}\} \in \Re^{n^2 \times n^2}$, and $V_2 = diag\{\alpha_{k11} \quad \cdots \quad \alpha_{k11}\} \in \Re^{nm \times nm}$

Furthermore, the state feedback matrix of each rule is described by

$$F_k = Z_k X^{-1} \tag{14}$$

The uncertain chaotic system (1) is quadratically stabilizable and the H^∞ control performance of (9) is guaranteed for a prescribed ρ^2.

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