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Controlling a Time-Varying Unified Chaotic System via Interval Type 2 Fuzzy Sliding-Mode Technique

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Chaos in control systems and controlling chaos in dynamical systems have been intensively studied over the past two decades. Nowadays, various techniques and methods have been proposed to achieve controlling chaos, such as synchronization of different chaotic system, adaptive control methods, back-stepping design techniques, and fuzzy sliding-mode control methods. In this study, we propose an interval type-2 fuzzy sliding mode control (IT2-FSMC) to control a time-varying unified chaotic system. This technique is a combination of the interval type-2 fuzzy logical control (IT2-FLC) method and the sliding mode control (SMC) approach and inherits the advantages of these two methods. The main contributions of this paper are: i) the decoupled IT2-FSMC is proposed for controlling the time-varying unified chaotic system; ii) the asymptotical stability can be achieved in the sense of Lyapunov stability criterion.

Consider a unified chaotic system which unites the classical Lorenz chaotic system, the Lü chaotic system, and the Chen's chaotic system, described below

$$\begin{cases} \dot{x}_1 = (25\alpha+10)(x_2-x_1) \\ \dot{x}_2 = (28-35\alpha)x_1 - x_1x_3 + (29\alpha-1)x_2 \\ \dot{x}_3 = x_1x_2 - (\alpha+8)x_3/3 \end{cases} \quad (1)$$

where $\alpha \in [0,1]$. The system (1) is the Lorenz system for $\alpha \in [0,0.8)$, the Chen system for $\alpha \in (0.8,1]$, and the Lü system for α=0.8

The controlled unified chaotic system can be expressed as

$$\begin{cases} \dot{x}_1 = (25\alpha+10)(x_2-x_1)+u_1 \\ \dot{x}_2 = (28-35\alpha)x_1 - x_1x_3 + (29\alpha-1)x_2 + u_2 \\ \dot{x}_3 = x_1x_2 - (\alpha+8)x_3/3 + u_3 \end{cases} \quad (2)$$

where u_i, $i = 1,2,3$ are the controllers to be designed. The tracking problem of (2) is to find a suitable control law to let the states follow a desired trajectory $x_d(t)$ asymptotically. The tracking error of the state vector is defined as

(3)

$$e(t) = x_d(t) - x(t) = [e_1 \quad e_2 \quad e_3]^T$$

where $e_i = x_{di} - x_i$, $i = 1, 2, 3$. Since each of the subsystems is controlled by an individual control variable u_i, we may design 3 decoupled sliding surfaces $s = [s_1 \quad s_2 \quad s_3]^T$ for the controllers $u = [u_1 \quad u_2 \quad u_3]^T$ as:

$$s_i = (\frac{d}{dt} + \lambda_i)e_i, \quad i = 1, 2, 3 \tag{4}$$

where $\lambda_i > 0$. Since (4) is a Hurwitz polynomial, maintaining system states on $s_i(e_i, t)$ will result in $e_i(t)$ approaching zero with the initial condition $e_i(0) \neq 0$. Consequently, the tracking control is fulfilled.

For each subsystem, a suitable control rule u_i has to be found so as to keep the error $e_i(t)$ on the sliding surface $s_i(e_i, t)$. To achieve this purpose, a positive Lyapunov function V is defined as

$$V = \frac{1}{2} s^T s \tag{5}$$

A sufficient condition for the stability of the system is given by

$$\dot{V} = \frac{1}{2}\frac{d}{dt}(s^T s) \le -\eta |s| \tag{6}$$

If we choose η>0, equation (6) states that the system is driven to the sliding surface as the absolute distance from $s_i(e_i, t)$ is diminishing.

If the state trajectory has reached the sliding surface $s_i = 0$, then from (4) one obtains $s_i = \dot{e}_i + \lambda_i e_i = 0$, $i = 1, 2, 3$ which implies that

$$\lim_{t \to \infty} e_i(t) = \lim_{t \to \infty} e_i(0) e^{-\lambda_i t} = 0 \tag{7}$$

An interval type-2 fuzzy set (IT2-FS) $\tilde{A}$ can be expressed as

$$\tilde{A} = \int_{x \in X} \int_{v \in J_x} 1/(x, v) = \int_{x \in X} \left[\int_{v \in J_x} 1/v \right] \Big/ x, \quad J_x \subseteq [0, 1] \tag{8}$$

The IT2-FLC contains the following five components: fuzzifier, rule base, fuzzy inference engine, type-reducer, and defuzzifier.

The fuzzifier maps crisp inputs into IT2-FSs. Here, the Gaussian shape MF with uncertain mean is chosen, and then the upper bound of the primary MF with uncertain mean can be expressed as

$$\bar{\mu}_{\tilde{A}}(x) \equiv \overline{FOU(\tilde{A})} = \begin{cases} e^{-\frac{1}{2}((x-m_1)/\sigma)^2}, & x < m_1 \\ 1, & m_1 \le x \le m_2 \\ e^{-\frac{1}{2}((x-m_2)/\sigma)^2}, & x > m_2 \end{cases} \tag{9a}$$

and the corresponding lower bound MF is

$$\underline{\mu}_{\tilde{A}}(x) \equiv \underline{FOU(\tilde{A})} = \begin{cases} e^{-\frac{1}{2}((x-m_2)/\sigma)^2}, & x \le \frac{m_1+m_2}{2} \\ e^{-\frac{1}{2}((x-m_1)/\sigma)^2}, & x > \frac{m_1+m_2}{2} \end{cases} \tag{9b}$$

where $[m_1,m_2]$ is the span of the uncertain mean and σ is the standard deviation.

The rule base in the IT2-FLC is also characterized by IF–THEN rules. We consider an IT2-FLC as having two inputs, e_i and $\dot{e}_i$, and a single output, u_i, then, the *j*-th rule of the IT2FLC for the control input u_i can be written as

$$R^j : \text{IF } e_i \text{ is } \tilde{F}_1^{ij} \text{ and } \dot{e}_i \text{ is } \tilde{F}_2^{ij} \text{ THEN } u_i \text{ is } \tilde{G}^{ij}, \quad j = 1, \cdots, M \tag{10}$$

Here we select the diagonal-type rule table due to the similarity between the diagonal type FLC and SMC, then one can redefine the diagonal in terms of an SMC with boundary layer, and verify its stability and robustness.

The inference engine combines all the fired rules and gives a non-linear mapping from the input IT2-FS to the output IT2-FS. The fuzzy rule in (12) can be written as

$$R^j : \quad \tilde{F}_1^{ij} \times \tilde{F}_2^{ij} \rightarrow \tilde{G}^{ij} \tag{11}$$

which results in an interval set described by their associated upper bound and lower bound for the *j*-th rule of control input u_i

$$\tilde{F}^{ij}(e_i, \dot{e}_i) = \left[\underline{f}^j(e_i, \dot{e}_i), \bar{f}^j(e_i, \dot{e}_i) \right] = \left[\underline{f}^j, \bar{f}^j \right] \tag{12}$$

The type-reducer is an extension version of the type-1 defuzzifier by applying the Extension Principle to a specific defuzzification method. The COS type-reduction is adopted here and the resulting type-reduced set can be expressed as

$$U_{\cos}(\boldsymbol{e}) = [u_l, u_r] = \int_{u^1 \in [u_l^1, u_r^1]} \cdots \int_{u^M \in [u_l^M, u_r^M]} \int_{f^1 \in [\underline{f}^1, \bar{f}^1]} \cdots \int_{f^M \in [\underline{f}^M, \bar{f}^M]} 1 \Big/ \frac{\sum_{i=1}^{M} f^i u^i}{\sum_{i=1}^{M} f^i} \tag{13}$$

where $U_{\cos}(e)$ is an interval output set determined by its left-most point u_l and its right-most point u_r. In order to compute u_l and u_r, the Karnik–Mendel iterative procedure is needed.

We defuzzify the interval set by using the average of u_l and u_r; hence, the defuzzified crisp output becomes

$$u_{fuzz\,2}(e) = \frac{u_l + u_r}{2} \tag{14}$$

The main advantage of the FSMC is that the number of rules can be reduced dramatically in comparison with the traditional FLC. Now, the decoupled sliding function $s_i(e_i, \dot{e}_i)$ is the only input of the proposed IT2FSMC and the *j*-th rule for *i*-th control input can be described as

$$R^j : \text{IF } s_i \text{ is } \tilde{S}^{ij} \text{ THEN } u_i \text{ is } \tilde{U}^{ij}, \quad j = 1 \to M \tag{15}$$

The outputs of the IT2FSMC $u_{ifsmc}(s_i)$, i=1,2,3 satisfy the following condition

$$|s_{ia}| > |s_{ib}| \to |u_{ifsmc}(s_{ia})| > |u_{ifsmc}(s_{ib})| \tag{16}$$

which means that the longer the distance of *s* is, the larger amplitude the control action will have.