On the evolution and run-up of breaking solitary waves on a mild sloping beach

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Tsunami calamity is one of the powerful natural disasters within the ocean environment. Field survey evidence indicates that the destruction has a direct relevance to the run-up and run-down motions upon a nearshore beach (Fig. 1a). Particularly, solitary-type long waves have been traditionally employed to study various tsunami behaviors owing to simulation simplicity and similarity of wave hydrodynamics. This report presents new laboratory experiments carried out in a supertank (300 m×5 m×5.2 m) (Fig. 1b) at Tainan Hydraulics Laboratory (THL) of National Cheng Kung University for simulating breaking solitary waves evolution and run-up on a 1:60 impermeable plane beach. Such a tank is capable of mimicking the large-scaled solitary waves propagating toward shallow water region, which gives valuable information close to a real tsunami-like wave. Sets of laboratory data of wave crest evolution and maximum run-up height are systematically measured and utilized to re-examine the available formulae given in previous literature. A simple formula for reasonably predicting the maximum run-up height of a breaking solitary wave on a plane beach with a wide range of beach slope (1:15-1:60) is also suggested.

Figure 1 (a): Field survey evidence of tsunami disaster upon nearshore beaches (provided by Prof. P. Lynett, Texas A&M Univ.); (b): Laboratory image of a solitary-wave-type tsunami propagating toward a 1:60 sloping beach in the present supertank (offshore view).

There are 54 cases performed in the experiments. The elevation of local water surface was recorded by employing 80-92 capacitance-type wave gauges located between 24 and 260 m downstream of the wavemaker. The maximum run-up heights on the slope were determined both from visual observation and run-up sensor. Additionally, the corresponding locations where the wave breaking commences were evaluated based on the mark on the sidewall compared with the gauge data. The obtained data are employed to re-examine the available formulae given in literature. It is noted the following only presents the results comparisons and more experimental descriptions are
referred to Hsiao et al. (2008).

Figure 2 Comparison of predictions by Grilli et al. (1997) (red line) and laboratory data (symbols), in which \( S_0 \) is the slope parameter, \( H_0 \) is the offshore wave height, \( h_0 \) is the offshore water depth, \( H_b \) and \( h_b \) are the wave height and water depth at the breaking point, respectively. SP: spilling breaker, PL: plunging breaker, SU: surging breaker.

Figure 2 shows the comparison between empirical formulae of Grilli et al. (1997) and laboratory data. The results suggest that (1) determinations of breaking point using visual observation and gauge data in a supertank are reasonable (Fig. 2a), (2) the formulae proposed by Grilli et al. are capable of predicting a breaking solitary wave on a mild slope (Figs. 2b and 2c). Particularly, the data (Fig. 2c) also reveal that the ratio of breaking wave height to breaking water depth is kept almost a constant value of 1.1 except for small wave steepness (i.e. larger \( S_0 \)).

Synolakis and Skjelbreia (1993) (hereafter named SS) noted that the wave amplitude evolution of a breaking solitary wave on a sloping beach can be divided into four zones for both PL or SP breakers. Particularly, in each zone the maximum local wave elevation can be mathematically described by a power-law-type formula shown in Eq. (1)

\[
\eta_{\text{max}} \sim \left( \frac{h}{h_b} \right)^n
\]

where \( \eta_{\text{max}} \) is the wave amplitude, \( h \) is the local water depth, \( n=-1/4 \) for the zone of gradual shoaling, \( n=-1 \) for the zone of rapid shoaling, \( n=4 \) for the zone of rapid decay and \( n=1 \) for the zone of gradual decay. Moreover, they also conjectured that the inner surf zone can be classified into more than two regions for the amplitude distributions on a plane beach with a bed slope less than 1:50. However, they cannot draw a decisive conclusion because of the lack of laboratory information. Figure 3 shows wave crest evolution of two typical wave nonlinearities on a mild sloping beach. Evidently, there are abundant data composed of five regions inside and outside the surf zones for both cases. Good agreements between data and Eq. (1) suggest that the formula by SS is capable of describing the wave crest evolution of a breaking solitary wave upon a mild slope. Interestingly, the data not only confirm that the decay zones proposed by SS are applicable but also demonstrate a further decay region after the gradual decay.
zone, in which the normalized wave height decreases approximately in proportion to a quarter power of the
dimensionless local water depth (i.e. $n=1/4$).

![Graph](image1)

Figure 3 Evolution of wave amplitude of a breaking solitary wave climbs up a 1:60 sloping beach. [(a): $e = 0.041$ ($\triangle$); $e = 0.052$ (+), (b): $e = 0.322$ ($\times$); $e = 0.338$ (●)].

![Graph](image2)

Figure 4 further shows the maximum run-up heights $R$ obtained in the present experiments. Note that the
maximum run-up height is defined as the maximum vertical distance from the run-up tongue of water shoreline to
the still water level. For the sake of comparison, the available laboratory data (see the caption of Fig. 4) are also
shown to analyze and it is also worthy to emphasize that all the laboratory data in Fig. 4 are relevant to breaking
solitary waves on different slopes. Evidently, the laboratory results in Fig. 4 exhibit that for a given slope the run-
up height increases as the growth of wave nonlinearity while the run-up height decreases as the reduction of slope
angle for a specific wave nonlinearity. In particular, a simple formula for predicting the maximum run-up height
based on the nonlinear least-square method and the laboratory data listed in Fig. 4 is empirically obtained as Eq. (2)

$$
\frac{R}{h_o} = 7.712 (\cot \beta)^{-0.632} (\sin \varepsilon)^{0.618}, \quad 0 < \varepsilon \leq 0.5
$$
Fig. 4 Normalized run-up height versus wave nonlinearity of breaking solitary waves on a plane slope.
[Experimental data of 1:15 slope by Li and Raichlen (2002)(×); experimental data of 1:19.85 slope by Synolakis (1987)(+); experimental data of 1:30 slope by Briggs et al. (1995)(*); present experimental data of 1:60 slope (●); Eq. (2) (red line)].

Reasonable agreements between the predictions of Eq. (2) and the laboratory data are obtained, indicating that the present formula is capable of estimating the reasonable run-up heights of braking solitary waves on a sloping bed with a wide range of beach slopes (i.e. $15 \leq \cot \beta \leq 60$). More comparisons are referred to Hsiao et al. (2008).

REFERENCES