Cloak for curvilinearly anisotropic media in conduction

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We explore the possibility to cloak a region in curvilinearly anisotropic background materials in the context of conductivity.

Recently the problem of cloaking has been treated in a number of publications. It was shown that by enclosing an object by an annulus with properly designed materials, the electromagnetic field can be controlled, passing around the object and returning undisturbed to their original trajectory, as though the object was not present. The key feature to achieve invisibility is the form invariance of the governing equation under curvilinear transformations. Since then there has been significant progress on cloaking objects to arbitrary fields. All researches on cloaking reported so far have focused on the background material which is isotropic or rectilinearly anisotropic. In this letter we explore the possibility of cloaking for curvilinearly anisotropic background materials. We focus on the physical phenomena of conductivity. The cylindrically orthotropic and spherically transversely isotropic conductivities of background can be denoted as

\[ \sigma_c = \text{diag}(\sigma_r, \sigma_\theta, \sigma_z) \]

and

\[ \sigma_s = \text{diag}(\sigma_A, \sigma_T, \sigma_T) \]

respectively. To cloak a region in curvilinearly anisotropic background material, a major distinction with the rectilinearly anisotropic material is that the cloak center \( \hat{O} \) is not the same with that of the material coordinate \( O \) (Fig. 1). Here we employ the previous transformation procedure together with a rigid-body translation to obtain the material property. In addition to the spatially varying and anisotropic properties, we see that the transformed conductivity tensor also depends on the location of cloak center and background material.

![FIG 1. A schematic representation of the coordinate system. Left figure is the original coordinate, and the right figure is the transformed coordinate](image)

To demonstrate the validity of the designed cloak with the center locate on (0.5, 0.5), we consider the inner radius as \( a = 0.25 \text{m} \) and the outer radius is \( b = 0.5 \text{m} \). The values of the conductivity tensor are taken to be \( \sigma_r = 400 \text{W/m}^{-1} \text{K}^{-1} \).
\( \sigma_\theta = \sigma_z = 100 \text{W/m-K} \) for the cylindrically orthotropic material to demonstrate the numerical calculations. Figure 2 (a) shows the potential field of the background material. Unlike the isotropic one, the contours of temperature display curved lines in a cylindrically orthotropic medium. By checking with analytical solutions, the potential distribution agrees well. Figure 2(b) illustrates the potential field of the same background containing a cylindrical cloak inside. Comparing with these two figures, the potential fields outside of the cloaking region remain undisturbed. For the numerical calculation, we assume \( \sigma_T = 400 \text{W/m-K} \) for the spherically transversely isotropic material, and the cloak was taken as a ball-like constituted by the cloak material properties. The size of the cloak is the same as those in cylindrical case and the center of cloak is placed at (0.5, 0.5, 0.5).

Figure 3(a) and Figure 3(b) show the potential field of the cloak on the outer boundary distributes non-uniformly but the potential fields on the inner boundary keep constant. The fields outside the cloak remain undisturbed, and the fluxes are bent around the inner sphere when they are getting into the cloak. After passing through the cloak, the fluxes return to their original trajectories without changing direction and magnitude.

In summary, using a modification of the coordinate transformation method, we design a cloak in conduction for cylindrically orthotropic and spherically transversely isotropic media. The design strategy reports a generalization of conventional cloak designs towards the design of cloaks in which the cloak center and the material origin are
not collocated. The concept may be further generalized to explore the possibility of cloak in certain heterogeneous or composite media.

REFERENCES